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Now the sum of any number of terms of a series $\Sigma (\sin n\pi z)/n$ is known to remain uniformly bounded,² and the series is known to converge everywhere, uniformly save in the immediate vicinity of the values $z = 0, \pm 2\pi, \pm 4\pi$, etc. Hence the series displayed above converges uniformly for all x, y save in the immediate vicinity of $x = y$, where, however, the sum of any number of terms of the series is bounded. Hence all six types of series will converge to values small numerically for $|\sigma - \bar{\sigma}|$ small, and, when multiplied through by any continuous function may be integrated term by term as to x , yielding uniformly convergent series in y . Thus the H series will have the stated properties for $|\sigma - \bar{\sigma}| \leq \delta > 0$.

3. *Generalization.*—The theorem suggests at once a theorem in General Analysis as defined by E. H. Moore.³ If we employ a quasi-geometrical terminology this generalization may be stated as follows: *any set of orthogonal vectors in a functional space lying near enough to a complete set of orthogonal vectors in that space is itself complete.* Another still wider generalization suggests itself: *any set of vectors in a functional space lying near enough to a complete set of vectors admitting a reciprocal set is itself complete and admits a reciprocal set.*⁴ This second generalization evidently plays the same part in relation to biorthogonal sets that the first does for orthogonal sets.

¹ See Kneser, *Die Integralgleichungen und ihre Anwendungen in der mathematischen Physik*, Braunschweig, 1911, pp. 84–95.

² See D. Jackson, *Rend. Circ. Mat. Palermo*, 32, 1911, (257–262).

³ See *Bull. Amer. Math. Soc., New York*, 18, 1911–1912, (334–362).

⁴ For a theorem of this type see an article of mine, *Paris, C.-R. Acad. Sci.*, 161, 1917, (942–945).

LOW-TEMPERATURE FORMATION OF ALKALINE FELDSPARS IN LIMESTONE

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Recent monographs by A. Heim¹ and D. Trümpy,² dealing with certain rock formations in Switzerland, put new emphasis on an important problem in minerogenesis. At different horizons of the Jurassic limestones of the Churfirsten-Mattstock mountain group, Heim has found abundant crystals of albite which have evidently developed *in situ* and are not of clastic origin. The crystals are automorphic, with maximum lengths of 0.2 mm. and average lengths much less (see fig. 150 in Heim's memoir). Minute crystals of ankerite are associated. Both albite and ankerite are regarded by Heim as due to crystallization on the sea-floor, during

the deposition of the limestone mud, which he interprets as likewise a chemical precipitate. The calcareous matrix has, in fact, the uniform, exceedingly fine grain characteristic of chemically precipitated limestone, though it encloses fragments of fossil shells and skeletons.

Trümpy, in the western Rhätikon, found similar albites, new crystallizations, in Jurassic limestone. These crystals are generally less than 0.03 mm. long and seldom as much as 0.05 mm. long. They are associated with colorless to somewhat brownish foils of mica with the same order of dimensions. Trümpy also describes secondary albite and orthoclase crystals, 0.05 to 0.08 mm. long, in the Tertiary (Flysch) limestones of the same district. The feldspars enclose roundish granules of calcite identical in size and form with the normal constituents of the mud-like limestone. In many cases the feldspar is merely a 'Bindemittel' for such grains of calcite. Trümpy agrees with Heim that the feldspars grew, 'schwebend,' in the calcareous mud.

Both Heim and Trümpy found no difficulty in distinguishing these new feldspars from the quite different clastic feldspars occasionally seen in the same thin sections. They have good evidence that the new feldspars were not developed by dynamic or contact metamorphism of the limestones—a conclusion borne out by the extremely fine grain of the calcareous material.

Thirty years before, Kaufmann³ described similar albites, "doubtless formed *in situ*", as occurring in Jurassic, Lower Cretaceous, Upper Cretaceous, and Eocene limestones of the "Emmen- und Schlierengegenden" of Switzerland. The maximum length of the albite crystals in any of the formations is 0.08 mm. Kaufmann noted that they abound specially in the purer limestones.

Independently Lory⁴ had found non-clastic albite and orthoclase crystals in most of the non-metamorphosed Jurassic limestones of the French Alps. Such feldspars are described as particularly abundant in nodular and geodic concretions. Pyrite crystals and minute, doubly-terminated quartz crystals accompany the feldspars. All the crystals are explained as growths in limestone muds charged with organic (bituminous) matter. A year later Lory⁵ noted the occurrence of albites and bipyramidal quartzes in Eocene limestone at Montricher, near Saint Jean-de-Maurienne. He attributed the formation of the feldspars to special conditions which were not stated, but he excluded metamorphism as an explanation. He found orthoclase crystals in the cast of an Ammonite.

Foullon⁶ reported the fine-grained to dense Eocene limestone of the Aegean island of Rhodes to contain 1.1 to 1.2% of well crystallized sili-

cates, including very pure albite. The largest feldspars seen were from 2.5 to 3.0 mm. in length. He suggested that the albite crystals originated, as tiny individuals, at the contact of sea-water and the calcareous mud at the sea-bottom; and that these grew to present size in the loose sediment as deposition went on. Only on such a hypothesis, he held, could one explain the automorphic character of the crystals. Since "certainly considerable time was necessary for their formation before being completely enclosed in the mud," he regards these crystals as proving the slowness of the calcareous precipitation. Foullon notes the absence of appreciable lime in the feldspar molecule itself.

Cayeux⁷ stated that similarly authigenic⁷ orthoclase is to be found in all the named horizons of the Chalk of the Paris basin. The crystals are always minute, averaging 0.04 to 0.05 mm. With constant crystal forms, they occur singly, never in groups. It is observed that orthoclase and glauconite tend to exclude each other in the various limestones. Cayeux assumes a community of origin for both minerals, described as "secondary." After examining his material, Michel Lévy, Lacroix, and Termier all agree that the orthoclase must have formed *in situ*.

Grandjean⁸ studied the chalk of Meudon, near Paris, concluding that its feldspar (probably microcline, rather than orthoclase) was formed on the floor of the Cretaceous sea, "contemporaneously with the deposition of the chalk." The feldspar "grew rapidly; it probably ceased to grow after it was once buried." He continued: "it is thoroughly [infinite] probable that feldspar is forming on the present sea-bottom."

A review of these European studies leads to the following generalizations:

1. Authigenic alkaline feldspars appear in non-metamorphosed limestones of Triassic, Jurassic, Cretaceous, and Eocene age and specially characterize many horizons in the Alpine Jurassic and Paris-basin Cretaceous.

2. Albite or orthoclase may occur alone or they may be associated in the same layer of limestone. Pyrite, bituminous matter, and bipyramidal quartz are not uncommon companions of the feldspars. One Jurassic limestone exhibits authigenic mica.

3. These feldspars in the European limestones typically appear as single crystals, with good crystal form and separated by the calcareous matrix; their lengths range generally from 0.03 to 0.08 mm.

4. All the calcareous matrices have the grain and texture of chemical precipitates. However, embedded in those originally muddy materials are the strongly contrasted fragments of organic origin, shells and skeletons.

5. All ten of the authors who have reported on the rocks mentioned agree that the albite or orthoclase has crystallized *in situ* without any help from metamorphism. Foullon, Grandjean, Trümpy, and Heim definitely express the view that the feldspars developed, as *chemical precipitates on the sea-floor* before or immediately after the burial of the corresponding laminae of sediment by the younger laminae. According to Cayeux, Lory held the same belief for the orthoclase crystals of the Jurassic limestones. The consensus of opinion is, then, that the authigenic feldspars and quartz attained their full growth well before normal diagenesis was completed, that is, while the calcareous muds were still unconsolidated.

These European researches are important from several points of view. For students of sediments they indicate a group of facts and problems which have been almost wholly untouched in American laboratories. They seem to throw light on the origin of an important dolomite in the Rocky Mountains. Finally, they furnish proofs of the crystallization of albite and orthoclase at temperatures much lower than have generally been assumed as possible.

In order to complete the summary of the recorded cases, that of the Rocky Mountain dolomite may be noted. It appears to represent the only known American parallel to the peculiar European limestones. The dolomite, named the Waterton formation, was discovered in 1905 by the writer,⁹ at Waterton lake just north of the boundary between Montana and Alberta. The formation, at least 60 meters thick, is the lowest exposed member of the great Rocky Mountain geosynclinal and is overlain by 4000 meters of other sediments referred to the Beltian (and probably Cambrian) series. Many hundreds of meters of still younger strata overlay those rocks before the composite geosynclinal was finally upturned, in the Tertiary era. The whole sedimentary cover on the Waterton dolomite was probably more than 6000 meters in thickness; yet the dolomite is lacking in signs of metamorphism, even that of the static kind. Igneous metamorphism can be excluded with equal definiteness.

The dolomite is chiefly composed of carbonate grains, which average about 0.02 mm. in diameter and seldom reach 0.05 mm. in diameter. These grains either interlock or occur as sharp rhombohedra, representing the crystal form so characteristic of true dolomite. Many laminae of the rock, up to 1 mm. or more in thickness, are charged with disseminated grains of glass-clear orthoclase, 0.01 to 0.05 mm. in diameter and without good crystal form. A few irregular grains of quartz, with similar dimensions, are usually associated. Other laminae are very rich in

clumps and lenticular masses of similar, minute crystals of orthoclase, which are interlocked and thus without crystal form (see fig. 7 of the writer's memoir). The clumps and lenses of feldspars regularly enclose many rhombohedra of dolomite. Occasionally in thin section one sees entire laminae, up to 1 or 2 mm. in thickness, composed of about equal parts of orthoclase and dolomite. The feldspar is clearly not of clastic origin.

Analysis of the rock gave the following result, which has been thoroughly checked, the alkalis being subjected to four closely accordant determinations by two different analysts.

SiO ₂	30.46	K ₂ O.....	5.77
Al ₂ O ₃	6.86	H ₂ O.....	1.42
Fe ₂ O ₃	4.53	CO ₂	22.55
FeO.....	1.89		
MgO.....	10.07		99.95
CaO.....	16.02	Sp. gr.....	2.749
Na ₂ O.....	0.38		

The estimated composition of the rock (assuming the iron oxides to represent magnetite) is, by weight:

	<i>per cent</i>		<i>per cent</i>
Orthoclase.....	34.5	MgCO ₃	21.2
Albite.....	3.1	CaCO ₃	28.6
Quartz.....	6.0		
Magnetite.....	6.3		99.7

The albite is probably in solid solution with the orthoclase. So far as known by published analysis no other non-metamorphosed dolomite or limestone even approaches the Waterton dolomite in its abundance of the alkalis; feldspar makes up about 40% of its volume.

When reporting on this singular rock the writer was not aware of the relevant observations of Lory, Foullon, Cayeux, and others. In 1912 the dolomite remained frankly a puzzle. The theoretical conclusions of Heim, Trümpy, and Foullon are therefore of value in suggesting a partial explanation of the still unique American rock. The Alberta formation differs from the European analogues in having a dolomitic base and in failing to exhibit good crystal form for the feldspar. This anhedral character may be connected with the specially high proportion of orthoclase, the numerous little crystals interfering with one another as they grew in the calcareous mud. The writer has not the slightest doubt as to the chemical origin of the carbonate material nor as to the authigenic nature of the orthoclase; he is inclined to refer the crystallization of the feldspar to the early period when the sediment was not yet consolidated nor deeply buried.

If Foullon, Grandjean, Trümpy, and Heim are correct in regarding these albite and orthoclase crystals as chemically precipitated *at* the sea floor, the formation temperatures must have been well below 100°C. If, on the other hand, the crystals developed only after burial of their calcareous matrices, the formation temperatures were nevertheless comparatively low. The Paris-basin rocks were never buried more than a few hundred meters. Assuming a burial of 1000 meters and the present thermal gradient (3° per 100 meters of depth), the formation temperatures could hardly have reached 100°. Though the burial was deeper for other cases (Alberta and the Alps), formation temperatures of much more than 200° cannot well be assumed, even if the crystallization were delayed until the maximum sedimentary covers were completed.

In any case the Paris-basin limestones apparently prove that albite, orthoclase, and quartz are crystallized in calcareous muds at temperatures lower than 100° and probably lower than 70°. Since this paper was practically completed for publication, it was found that Doelter¹⁰ had already assumed a formation temperature of 'perhaps 100°' for the potash feldspars of the Paris-basin Chalk.

The lowest recorded temperature at which orthoclase has been artificially prepared is 300°. That feat was accomplished by Chrustschoff,¹¹ who developed both orthoclase and quartz in an aqueous solution of dialyzed silica with alumina and caustic potash, kept for several months at the temperature mentioned. Friedel and Sarasin¹² using analogous solutions in bombs, produced albite, orthoclase, and quartz in much shorter periods, but they employed a temperature of 500°. These experiments suggest that time may be one of the important factors aiding the crystallization of alkaline feldspars at very low temperatures.

What the exact chemical conditions for these noteworthy precipitations in calcareous mud were, is a question now impossible to answer. Perhaps the abundance of lime in the muddy matrices affected the solubility of the alkalis in sea-water;¹³ one should further consider the possible influence of decaying animal matter on the various solubilities involved. The astonishing concentration of potash feldspar in the Water-ton dolomite accentuates the difficulty of the problem.

¹ Heim, A., *Beitr. geol. Karte Schweiz*, N. F., Lief 20, 1916, (514, 543, 561, 567).

² Trümpy, D., *Ibid.*, Lief., 46, 1916, (83, 108).

³ Kaufmann, F. J., *Ibid.*, (Ser. 1), Lief., 24, 1886, (583).

⁴ Lory, C., *Paris, C.-R. Acad. Sci.*, 103, 1886, (309).

⁵ Lory, C., *Ibid.*, 105, 1887, (99).

⁶ Foullon, H. B., *Sitzber. Wiener Ak.*, 100, Abt., 1, 1891, (162, 169).

⁷ Cayeux, L., *Paris Mém. Soc. Géol.*, 4, No. 2, 1897, (259, 279, 303, 432-434). F. Grandjean, *Paris, C.-R. Acad. Sci.*, 148, 1908, (723), holds that these potash feldspars described

by Cayeux have the optical properties of microcline. Termier found authigenic albite crystals in the Flysch limestone of Briançonnais. Compare Rosenbusch, H., *Elemente der Gesteinslehre*, 3te Aufl., Stuttgart, 1910, (520). "Authigenic" means "formed *in situ*."

⁸ Grandjean, F., *Paris, C.-R., Acad. Sci.* 148, 1908, (723).

⁹ Daly, R. A., *Geol. Surv. Can., Memoir* No. 38, 1912, (50).

¹⁰ Doelter, C., *Handbuch der Mineralchemie*, Dresden, Bd. 2, 2te Hälfte, 1915, (556).

¹¹ Chrústschoff, K., *C.-R. Acad. Sci.*, 104, 1887, (602).

¹² Friedel, C., and Sarasin, E., *Ibid.*, 92, 1881, (1374).

¹³ Cf. Zschimmer, E., in C. Doelter's *Handbuch der Mineralchemie*, 1, 1912, (910).

THE INTERFEROMETRY OF SMALL ANGLES, ETC. METHODS BY DIRECT AND REVERSED SUPERPOSED SPECTRA

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1. *Introductory*.—It occurred to me that a number of the methods treated in my papers on direct and reversed spectrum interferometry might be used directly for the measurement of small angles and possibly of the distance of the source of light. Such a procedure would have an apparent advantage, at least theoretically, of not calling for the preliminary superposition of the images of distant objects, as the superposition is inherent in the method itself. But there are large constants involved which make the result very problematical, unless these constants can be removed by a compensator. It is very questionable, moreover, whether, appreciable interferences can occur, and another difficulty which hampers the method is the decrease in the size of objects as their distances increase. A progressive investigation with the object of ascertaining to what degree the experiment is feasible is nevertheless worth while. It will be convenient therefore first to develop the methods without reference to the ulterior conditions which limit the interferences and this method has been pursued.

2. *Method with Prism*.—Figure 1 is a sketch of one of the methods in which S is the distant source of light, from which rays d and d' strike the mirrors m and n , are thence reflected to the silvered sides of the right angled prism P . After leaving it the rays enter the spectro-telescope at T in parallel. If the proper angles are selected the prism P may be replaced by one of any angle or by a reflecting grating.

Suppose now the system mPn is securely attached to a rigid metallic beam or rail capable of rotating around a vertical axis at its center (P). This is indicated in figure 2 where the direction of rays and the normals

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